Some Theorems on the Relation between Profitable Speculation and Price Stability*

Jörg Schimmler

Introduction

In the discussion about the influence of speculative activities on prices in a competitive market FRIEDMAN's assertion(1) that profitable speculation always stabilizes price fluctuations -the so-called FRIEDMAN Theorem - plays a prominent role. Although the validity of this proposition would not imply that markets with speculative activities always display lower price fluctuations, various implications for economic policy are to be expected. This may be one reason for the proposals to limit amateur speculation in favour of professional speculation since the latter seems to be more often profitable than the former(2). On the basis of the model outlined by TELSER(3), KEMP(4) proves that a linear non-speculative excess demand function implies the

*The present article has substantially benefitted by many valuable suggestions and comments of Dr. K. Floret. I am grateful to Prof. M. J. Farrell for a stimulating correspondence on the subject of this paper. Errors and shortcomings are entirely the responsibility of the author.
(1)M.FRIEDMAN, Essays in Positive Economics. Chicago 1953, p.175. This argument appears to go back to MILL. See J.S.MILL, Principles of Political Economy. With Some Applications to Social Philosophy. Vol.2, 6th Ed., London 1865, p. 279.
(2)The empirical results of HOUTHAKKER seem to support this hypothesis.See H.S.HOUTHAKKER, Can Speculators forecast Prices? "The Review of Economics and Statistics", Cambridge, Mass. Vol. 39(1957), p. 143-151.

validity of FRIEDMAN's proposition. The necessity of this condition is shown by FARRELL(5) assuming that there is no temporal interdependence between speculative net sales and the price variations.

These results remain valid when FARRELL's independence assumption is substantially weakened. If no transactions costs are considered temporal interdependence in the non-speculative excess demand function implies that FRIEDMAN's theorem fails to hold(6). The great importance of transactions costs for the problem under consideration is shown by FARRELL(7). Assuming the temporal independence mentioned above he deduces sufficient conditions for the validity of FRIEDMAN's assertion.

Until recently(8), to the best of the author's knowledge, no mathematical analysis of temporal interdependence and transactions costs in respect to speculation and price stability was available. The objective of the present study is to give a characterization of those market reactions(9) which are

(3)L.G.TELSER, A Theory of Speculation Relating Profitability and Stability. "The Review of Economics and Statistics", Cambridge, Mass., Vol. 41 (1959), p. 295-301.
(4)M.C.KEMP, Speculation, Profitability, and Price Stability. "The Review of Economics and Statistics", Cambridge, Mass., Vol.45(1963), p. 185-189.
(5)M.J.FARRELL, Profitable Speculation. "Economica", London, N.S., Vol. 33(1966), p. 183-193.
(6)J.SCHIMMLER, Speculation, Profitability, and Price Stability - a Formal Approach. "The Review of Economics and Statistics", Cambridge, Mass., Vol. 51(1973), p. 110-114.
(7)FARRELL,op. cit.
(8)See J.SCHIMMLER, Spekulation, spekulative Gewinne und Preisstabilität. Eine Theorie der Spekulation unter besonderer Berücksichtigung der Auswirkungen spekulativer Transaktionen auf die Preisstabilität. Meisenheim 1974.
(9)The market reaction is the inverse non-speculative excess demand function used by FARRELL. This notion will be defined below.

compatible with FRIEDMAN's hypothesis. The influence of transactions costs on the validity of this proposition is analyzed and further details are given in terms of the correlation between speculation and market reaction.

Definition of the Model and Notation

The model to be formulated in the sequel describes an abstract market for a commodity. We shall define it on the basis of TELSER's and FARRELL's framework(10). For the sake of simplicity we follow FARRELL in choosing a discrete time variable(11) $t=1,2,\dots,\tau$ such that all variables depending on time can be interpreted as vectors of the τ-dimensional Euclidean space $\mathbb{R}^{\tau}$. In the market in question it is assumed that a distinction between speculative and non-speculative transactions(12) allows a clear definition of speculative net purchases as the difference between purchases and sales of the aggregate speculators. This vector $S=(s_1,s_2,\dots,s_{\tau})$ will also be called a speculation.

For the market without speculation, i.e. $S=0$ the resulting prices form the vector P_w whereas the price vector in presence of a speculation S is denoted by P_s.

(10)See TELSER,op.cit. and FARRELL,op.cit.
(11)This would not mean a restriction of generality since a model with continuous time variables can simultaneously be established. Definitions and proofs can easily be transferred as pointed out in the appendix. See also SCHIMMLER, Spekulation,...,op. cit.
(12)This distinction is quite formal. Consequently, the labels "speculative" and "non-speculative" may considerably differ from the usual meaning of the words.

Price vectors are assumed to be positive(13). In general the set $\mathcal{P}$ of price vectors can be taken as the convex cone(14) $\mathcal{R}^+$ of all positive vectors in $\mathbb{R}^{\tau}$. In order to allow other choices of $\mathcal{P}$ the following properties of $\mathcal{P}$ may be given which hold for $\mathcal{R}^+$.

(1) $U \in \mathcal{P} \subset \mathcal{R}^+$ where U is the vector whose elements are unity.

(2) If $P \in \mathcal{P}$ and $X \in \mathbb{R}^{\tau}$ there is a positive number a such that for all $b>a$ the vector $bP+X$ belongs to $\mathcal{P}$.

In the following, sets having the second property will be called conically absorbent(15). In $\mathbb{R}^{\tau}$ this condition is equivalent for a convex cone $\mathcal{P}$ to be open.

The functional relationship between P_w, S, and P_s is given by(16)

$$P_s = P_w + H(S)$$

in which the so-called market reaction(17) H displays the influence of S on the price difference $P_s - P_w$ in the market under consideration.

The independence of H from P_w entails some problems which can be solved by some definitions(18). Because P_s is to be considered as positive for all price vectors P_w the vector $H(S)$ must be non-negative for all speculations. A model for a market in which any speculative activity leads to an increased price appears

(13)A vector is said to be positive if its elements are positive.
(14)For the definition of convex cones see appendix.
(15)This name goes back to a proposal of K.FLORET and may be regarded as provisional.
(16)See SCHIMMLER,Speculation,...,op.cit.,p.113.
(17)The market reaction is the inverse function of FARRELL's "non-speculative excess demand function".
(18)I am indebted to K.FLORET for focussing my attention to this problem.

somewhat degenerate.

This difficulty can be evaded by taking price vectors P out of a suitably selected subset $\tilde{\mathcal{P}}$ of $\mathcal{P}$ which contains only "sufficiently large" price vectors. Therefore we introduce the following condition: Let $P \in \tilde{\mathcal{P}}$ then there is a positive number a so that $bP \in \tilde{\mathcal{P}}$ for all $b \geq a$.

The example of a so-called proportional market reaction H(S)=aS shows that an appropriate domain $\mathcal{S}$ of H is to be selected in order to guarantee that P_s belongs to $\mathcal{P}$ for all $P_w \in \tilde{\mathcal{P}}$ and all $S \in \mathcal{S}$.

A simple example for a suitable choice of $\tilde{\mathcal{P}}$ and $\mathcal{S}$ in the case of the market reaction H(S)=bS is given by(19):
$\tilde{\mathcal{P}} = \{P: P \in \mathcal{P}, P(t) \geq a\}$ and $\mathcal{S} = \{S: |bS(t)| < a/2\}$ where a is some positive number.

In accordance with TELSER, KEMP, and FARRELL we consider speculative profits π exclusively in the form of realized profits. This is achieved by the requirement that the sum of speculative net purchases is zero or $S \cdot U = 0$ for all $S \in \mathcal{S}$ in which the dot denotes the common inner or scalar product in $\mathbb{R}^{\tau}$ and U is the vector already introduced. Thus the concept of realized speculative profits is equivalent to $\mathcal{S} \subset \mathcal{U}^{\perp}$ where $\mathcal{U}^{\perp} = \{X: X \cdot U = 0\}$ is the orthogonal complement to the subspace spanned by U. The economic consequences of this convention have been discussed elsewhere(20).

Leaving aside transactions costs, insurance, storage costs etc., speculative profits π are given by $\pi = -P_s \cdot S$. Losses are of

(19)See SCHIMMLER,Spekulation,...,op.cit.
(20)See TELSER, op. cit. and FARRELL, op. cit.

course negative profits. The minus sign in the definition of speculative profits is a consequence of the definition of S. Following FARRELL we assume two types of transactions costs for the aggregate speculators and define the total transactions costs by(21)

$$k_T(P_w, S) = (P_w + H(S)) \cdot T(S) + k(S) = P_s \cdot T(S) + k(S)$$

in which T is a mapping from $\mathcal{S}$ into the convex cone $\mathcal{C}^+$ of positive vectors in $\mathbb{R}^{\tau}$ and k is a positive real valued function on $\mathcal{S}$. As price vectors are assumed to be positive, the inner product of T(S) and P_s is positive. Therefore transactions costs are positive by definition. Speculative profits π can now be formulated as follows:

$$\pi = -P_s \cdot S - P_s \cdot T(S) - k(S).$$

Writing $\tilde{S} = S + T(S)$ we obtain

$$\pi = - P_s \cdot \tilde{S} - k(S).$$

In the present model the measure of stabilization c is defined by(22)

$$c = \mathrm{var}(P_w) - \mathrm{var}(P_s)$$

in which $\mathrm{var}(X) = (1/\tau)\|X - \bar{X}\|^2$ where $\bar{X} = (1/\tau)(X \cdot U)U$ and $\|X\|$ is the length of X.

The speculation S is said to have stabilized the price fluctuations (of P_w) when c>0. Thus in the model in question FRIEDMAN's proposition is valid if the implication

$$\pi > 0 \Rightarrow c > 0$$

(21)Although S is the vector of speculative net purchases it is assumed that transactions costs will be a function of S and P_s. Our definition of the transactions costs is a generalization of FARRELL's.
(22)The reasons for selecting this measure of stabilization have been discussed elsewhere.

holds for all $S \in \mathcal{S}$ and $P_w \in \tilde{\mathcal{P}}$. To exclude trivial cases, we assume in the following that for every $S \in \mathcal{S}$ there is a $P_w \in \tilde{\mathcal{P}}$ so that $\pi > 0$. Because $S = 0$ is of no interest for markets with speculative activities, the set $\mathcal{S}$ shall not contain the vector 0.

The validity of FRIEDMAN's proposition

Before formulating and proving the central Theorem of this section, some remarks should be made in order to obtain an efficient notation. Generally, X^* will denote the vector $X - \bar{X}$ which is the projection $Pr_{\mathcal{U}^\perp}(X)$ of X on $\mathcal{U}^\perp$. With this notation we get the following expressions for the measure of stability c which will be used interchangeably:

$$\begin{aligned} \tau c &= \tau(\mathrm{var}(P_w) - \mathrm{var}(P_s)) = P_w^* \cdot P_w^* - P_s^* \cdot P_s^* \\ &= (P_w^* - P_s^*) \cdot (P_w^* + P_s^*) = - H^*(S) \cdot (P_w^* + P_s^*) \\ &= - H^*(S) \cdot (2P_w^* + H^*(S)) = - H^*(S) \cdot (2P_s^* - H^*(S)). \end{aligned}$$

For all $S \in \mathcal{S}$ the non-empty set $\mathcal{Q}_s = \mathcal{P} \cap \{X : X \cdot S < - k(S)\}$ is conically absorbent which can easily be verified. As this property is invariant to linear mappings, in $\mathcal{U}^\perp$ the same is true for the projection $\mathcal{Q}_s^*$ of $\mathcal{Q}_s$ on $\mathcal{U}^\perp$.

The dual cone $(\mathcal{Q}_s^*)^d$ of $\mathcal{Q}_s^*$ in $\mathcal{U}^\perp$ is defined by $\{X : X \cdot Y \leq 0$ for all $Y \in \mathcal{Q}_s^*, X \in \mathcal{U}^\perp\}$. The set $(\mathcal{Q}_s^*)^d$ is a (closed) convex cone(23) which contains S. To show this, let $P^* \in \mathcal{Q}_s^*$ be an arbitrary vector. Then a vector $P \in \mathcal{Q}_s$ exists so that $Pr_{\mathcal{U}^\perp}(P) = P^*$. By definition of $\mathcal{Q}_s$ we have $- P^* \cdot S = - P \cdot S > P \cdot T(S) + k(S) > 0$.

(23)See appendix, proposition 1.

Therefore we obtain $S \cdot P^* < 0$ for all $P^* \in \mathcal{Q}_s^*$, i.e. $S \in \mathcal{Q}_s = (\mathcal{Q}_s^*)^d - \{0\}$ because $S \neq 0$. For the sake of brevity the half-space $\{X : X \cdot H^*(S) > 0, X \in \mathcal{U}^\perp\}$ is denoted by $\mathcal{H}_s^*$.

Theorem 1: FRIEDMAN's proposition is valid for the market reaction H if and only if $H^*(S) \in \mathcal{Q}_s$ for all $S \in \mathcal{S}$.

Proof: Let H be a market reaction so that $H^*(S) \in \mathcal{Q}_s$ for all $S \in \mathcal{S}$. If $\pi > 0$ then $P_s \in \mathcal{Q}_s$. By the definition of the dual cone $(\mathcal{Q}_s^*)^d$ we obtain $P_s^* \cdot H^*(S) \leq 0$ for all $P_s^* \in \mathcal{Q}_s^*$ and therefore

$$\tau c = - H^*(S) \cdot (2P_s^* - H^*(S)) = - 2P_s^* \cdot H^*(S) + \| H^*(S) \|^2 > 0$$

since $H^*(S) \neq 0$. Thus FRIEDMAN's Theorem is valid. Conversely, if there is a vector $S \in \mathcal{S}$ so that $H^*(S) \notin \mathcal{Q}_s$ and $H^*(S) = 0$, there will be no stabilization at all though the speculative profits are positive when $P_s \in \mathcal{Q}_s$. If $H^*(S) \notin (\mathcal{Q}_s^*)^d$ then there is a vector $X^* \in \mathcal{Q}_s^*$ so that $X^* \cdot H^*(S) > 0$, i.e. $X^* \in \mathcal{H}_s^* \cap \mathcal{Q}_s^*$. As $\mathcal{Q}_s^*$ is the projection of $\mathcal{Q}_s$ on $\mathcal{U}^\perp$ there is a vector $X \in \mathcal{Q}_s$ such that $Pr_{\mathcal{U}^\perp}(X) = X^*$. Now we can select a positive number a so that $aX \in \tilde{\mathcal{P}}$, $aX^* + H^*(S) \in \mathcal{H}_s^* \cap \mathcal{Q}_s^*$ and $aX + H(S) \in \mathcal{Q}_s$ hold simultaneously. This can be achieved by the definition of $\tilde{\mathcal{P}}$ and by the fact that $\mathcal{H}_s^* \cap \mathcal{Q}_s^*$ and $\mathcal{Q}_s$ are conically absorbent. Let $P_w = aX$. Then we have $P_s = aX + H(S) \in \mathcal{Q}_s$ and therefore $\pi > 0$. On the other hand

$$\tau c = - H^*(S) \cdot (2P_w^* + H^*(S)) = - 2aH^*(S) \cdot X - \| H^*(S) \|^2 < 0,$$

since $H^*(S) \cdot X > 0$. FRIEDMAN's proposition fails to hold for this case and thus the Theorem is proved.

It should perhaps be noticed that $H^*(S) \in \mathcal{Q}_s$ if and only if $H(S) \in \mathcal{Q}_s + \mathcal{U}$ where $\mathcal{Q}_s + \mathcal{U} = \{X : X = Y + Z, Y \in \mathcal{Q}_s$ and $Z = dU\}$.

When transactions costs are not taken into account, the set $\mathcal{G}_S^*$ is given by $\{X : X \cdot S < 0, X \in \mathcal{U}^\perp\}$ and therefore $\mathcal{G}_S$ equals the convex cone $\mathcal{K}(S)$ generated by S(24). We retain the known result (25) that FRIEDMAN's Theorem holds in the case of zero transactions costs if and only if $H^*(S) = b(S)S$ for all $S \in \mathcal{S}$ in which b is some positive real valued function on $\mathcal{S}$. Thus we obtain

$$P_S(t) - P_W(t) = b(S)S(t) + \bar{P}_S - \bar{P}_W \text{ for } t=1,2,\ldots,\tau.$$

FARRELL's independence assumption is shown to be a necessary condition for the validity of FRIEDMAN's proposition if no transactions costs are assumed. Any temporal interdependence in the market reaction would imply that FRIEDMAN's Theorem fails to hold. This situation does not change if only transactions costs of the form k(S) are assumed (26). To show this, we demonstrate that $\mathcal{G}_S$ equals the truncated convex cone $\mathcal{K}(S)$ with vertex 0 generated by S. In this case speculative profits π are given by

$$\pi = -P_S \cdot S - k(S) = -P_S^* \cdot S - k(S)$$

so that $\mathcal{G}_S^* = \{X : -X \cdot S > k(S), X \in \mathcal{U}^\perp\}$ whose dual cone in $\mathcal{U}^\perp$ is given by $(\mathcal{G}_S^*)^d = \{X : X = bS, b \geq 0\}$, yielding $\mathcal{G}_S = \mathcal{K}(S)$.

If we denote the set of all market reactions (27) and the subset of those which imply the validity of FRIEDMAN's Theorem by $\mathfrak{R}$ and $\mathfrak{R}_{\mathfrak{F}}$, respectively, we can establish the following theorem:

(24)See appendix, proposition 2.
(25)See SCHIMMLER, Spekulation,... op. cit.
(26)This is inconsistent with FARRELL's results embodied in his inequality (29). However, in correspondence with the author, FARRELL has stated that (29) was incorrect, and his corrected version is consistent with our results. See FARRELL, op.cit.,p.190.
(27)That are all mappings $H : \mathcal{S} \longrightarrow \mathbb{R}^\tau$ such that $P_W + H(S) \in \mathcal{P}$ for all $P_W \in \mathcal{P}$.

Theorem 2: If k_1 and k_2 are transactions costs such that $k_1(P_W,S) \leq k_2(P_W,S)$ for all $P_W \in \tilde{\mathcal{P}}$ and $S \in \mathcal{S}$ then for the appropriate sets of market reactions $\mathfrak{R}_{\mathfrak{F}}^1$ and $\mathfrak{R}_{\mathfrak{F}}^2$ the inclusion $\mathfrak{R}_{\mathfrak{F}}^1 \subset \mathfrak{R}_{\mathfrak{F}}^2$ holds(28).

Proof: Reviewing Theorem 1 it suffices to show that $\mathcal{G}_S^1 \subset \mathcal{G}_S^2$ for all $S \in \mathcal{S}$. Let $P \in \mathcal{G}_S^{*2}$ then by definition of $\mathcal{G}_S^{*2}$ the inequality $-P \cdot S - k_2 > 0$ holds. As $k_1 < k_2$ we get $-P \cdot S - k_1 > 0$. Thus we have $P \in \mathcal{G}_S^{*1}$ and therefore $\mathcal{G}_S^{*2} \subset \mathcal{G}_S^{*1}$ which implies $\mathcal{G}_S^1 \subset \mathcal{G}_S^2$.

Now we may formulate the following result:
Increasing transactions costs lead to a wider set of market reactions which imply the validity of FRIEDMAN's proposition. The expectation that profitable speculation stabilizes price fluctuations is a monotone increasing function of the transactions costs. A warning should perhaps be added. High transactions costs would lessen speculators' engagements and so would weaken the stabilizing effect. Also the influence of transactions costs on the relation between amateur and professional speculation should be taken into account.

The notion of the correlation coefficient allows to deduce some further results from Theorem 1. It is defined for vectors X,Y with non-vanishing projections X^*, Y^* on $\mathcal{U}^\perp$ by

$$\varrho(X,Y) = X^* \cdot Y^* / (\|X^*\| \|Y^*\|).$$

By virtue of the well known inequality of CAUCHY-SCHWARZ we have $-1 \leq \varrho(X,Y) \leq 1$ and $\varrho(X,Y) = 1$ if and only if $Y^* = aX^*$ for

(28)The index i=1,2 refers to k_1 and k_2, respectively.

some positive number a. If there are no transactions costs or the transactions costs are of the form k(S), Theorem 1 says that FRIEDMAN's proposition is valid if and only if

$$\rho(H(S),S) = 1$$

for all $S \in \mathfrak{T}$. In the general case we want to deduce sufficient conditions for the validity and non validity of FRIEDMAN's theorem. Therefore the following real valued functions on $\mathfrak{T}$ are introduced:

$$f(S) = \sup \{ \rho(X,S) : X^* \notin \mathfrak{A}_{\delta s} \}$$

$$g(S) = \inf \{ \rho(X,S) : X^* \notin \mathfrak{A}_{\delta s} \}.$$

The inequality $g(S) \leq f(S)$ holds(29) for all $S \in \mathfrak{T}$ and $\tau \geq 3$.

Theorem 3: If for all $S \in \mathfrak{T}$ the relation $\rho(H(S),S) > f(S)$ is true then H belongs to $\mathfrak{G}_{\delta F}$. If for one $S \in \mathfrak{T}$ the relation $\rho(H(S),S) < g(S)$ holds then $H \notin \mathfrak{G}_{\delta F}$.

Proof: If $\rho(H(S),S) > f(S)$ then $H^*(S) \in \mathfrak{A}_{\delta s}$ and if $\rho(H(S),S) < g(S)$ then $H^*(S) \notin \mathfrak{A}_{\delta s}$. An application of Theorem 1 yields the assertion.

Therefore g(S) is a lower bound for the correlation between H(S) and S in respect to all market reactions of $\mathfrak{G}_{\delta F}$, whereas f(S) is an upper bound for the correlation between H(S) and S of all market reactions not belonging to $\mathfrak{G}_{\delta F}$.

With the aid of Theorem 2 it is easy to prove that rising transactions costs entail decreasing bounds f(S) and g(S). Thus

(29) This inequality is not needed in the sequel. For a proof see SCHIMMLER, Spekulation,..., op. cit.,p.119.

more market reactions will be compatible with FRIEDMAN's theorem. Because of these properties of f and g the correlation between H(S) and S may perhaps provide a useful criterion for an empirical investigation in connection with the validity of FRIEDMAN's Theorem.

Appendix

As mentioned above, the model underlying our considerations can be as well established in terms of a continuous time variable(30). Let the domain of t be the closed interval $[0,\tau]$, where τ is some positive number. The time-dependent variables are real valued functions on $[0,\tau]$ which are assumed to be measurable and quadratic Lebesgue- integrable in forming the well known Hilbert space $\mathcal{L}^2_{[0,\tau]}$. In $\mathcal{L}^2_{[0,\tau]}$ a linear space $\mathcal{Z}$ of bounded functions containing $u(t) = 1$ is selected. The time-dependent variables are the elements of $\mathcal{Z}$ which now plays the role of $\mathbb{R}^\tau$. Let $\mathcal{A}^+$ be the set of all functions of $\mathcal{Z}$ with positive lower bound(31) then $\mathbb{P}$ is to be selected just in the same way. The inner product in $\mathcal{Z}$ is given by

$$X \cdot Y = \int_0^\tau x(t)y(t)dt.$$

which is the usual definition of the inner product in $\mathcal{L}^2_{[0,\tau]}$. The projection on a subspace $\mathcal{V}$ of $\mathcal{Z}$ is defined when $\mathcal{V}$ or its orthogonal complement $\mathcal{V}^\perp$ has a finite dimension. Throughout the present article only projections of this type are needed.

Convex sets and cones

Definition 1: A subset $\mathcal{M}$ of the linear space $\mathcal{Z}$ is said to be convex if whenever it contains X and Y it contains $aX + (1-a)Y$ where $0 \leq a \leq 1$.

(30) More details are found in SCHIMMLER, Spekulation,...,op.cit.
(31) $\mathcal{A}^+$ is not open but algebraically open. For the definition of algebraically open sets see G.KÖTHE, Topological Vector Spaces I. Berlin, Heidelberg, New York 1969, p. 177.

Definition 2: A subset $\mathcal{M}$ of the linear space $\mathcal{Z}$ is called a cone with vertex X_0 if it contains $X_0 + a(X - X_0)$ for all $a>0$ whenever it contains X. A cone is said to be truncated if it does not contain its vertex and to be pointed otherwise.

Definition 3: A subset $\mathcal{M}$ of the linear space $\mathcal{Z}$ is called conically absorbent in $\mathcal{Z}$ if for all $X \in \mathcal{M}, Y \in \mathcal{Z}$ there is a positive number a so that $bX + Y \in \mathcal{M}$ for all $b \geq a$.

We obtain the following immediate consequences of the definition of conically absorbent sets:

(1) Unions and finite intersections of conically absorbent sets are conically absorbent.

(2) If $\mathcal{M} \neq \emptyset$ is conically absorbent in $\mathcal{Z}$ then $\mathcal{M} - \mathcal{M} = \mathcal{Z}$.

(3) If $0 \in \mathcal{M}$ and $\mathcal{M}$ is conically absorbent then $\mathcal{M} = \mathcal{Z}$.

(4) If $f: \mathcal{Z} \longrightarrow \mathcal{Y}$ is linear and $\mathcal{M}$ conically absorbent in $\mathcal{Z}$ then $f(\mathcal{M})$ is conically absorbent in $f(\mathcal{Z})$.

An example of a conically absorbent set is the set $\mathcal{A}^+$ of the bounded functions of $\mathcal{L}^2_{[0,\tau]}$ with positive lower bound in the subspace of the bounded functions. An open convex cone with vertex 0 in the pre-Hilbert space $\mathcal{Z}$ is conically absorbent.

Definition 4: The dual cone of the subset $\mathcal{M}$ of (the pre-Hilbert space) $\mathcal{Z}$ in $\mathcal{Z}$ is the set $\mathcal{M}^d = \{X: X \cdot Y \leq 0 \text{ for all } Y \in \mathcal{M}, X \in \mathcal{Z}\}$.

Proposition 1: The dual cone of $\mathcal{M}$ in $\mathcal{Z}$ is a closed pointed convex cone with vertex 0.

<u>Proof:</u> Because $\mathcal{M}^d$ is the intersection of the closed pointed convex cones $\mathcal{H}_Y = \{X : X \cdot Y \leq 0, Y \in \mathcal{M}\}$ the assertion holds.

An obvious consequence of the definition of the dual cone is the implication $\mathcal{M} \subset \mathcal{N} \Rightarrow \mathcal{N}^d \subset \mathcal{M}^d$.

<u>Proposition 2:</u> The dual cone of $\mathcal{H}_{Y,a} = \{X : X \cdot Y > a, a > 0\}$ is the pointed cone generated by $-Y$.

Proof: If $Z \in \mathcal{K}(-Y)$ then $Z = -bY$ for some $b \geq 0$. We obtain for all $X \in \mathcal{H}_{Y,a}$ the inequality

$$X \cdot Z = -bX \cdot Y \leq -ba \leq 0.$$

thus $\mathcal{K}(-Y) \subset \mathcal{H}^d_{Y,a}$. If $Z \notin \mathcal{K}(-Y)$ then in the case of $Z = bY$ for some positive b, the dual cone $\mathcal{H}^d_{Y,a}$ does not contain Z. If Z and Y are linearly independent the determinant of the system

$$(qY + rZ) \cdot Y = a+1$$

$$(qY + rZ) \cdot Z = a+1$$

for q and r is non vanishing because $|Y \cdot Z|^2 < \|Y\|^2 \|Z\|^2$ by virtue of the inequality of CAUCHY-SCHWARZ. Therefore the vector $X = qY + rZ$ belongs to $\mathcal{H}_{Y,a}$ and yields $X \cdot Z > 0$. Thus Z does not belong to $\mathcal{H}^d_{Y,a}$.

References

FARRELL, M.J., Profitable Speculation. "Economica", London, N.S., Vol. 33 (1966), p. 183-193.

FRIEDMAN, Milton, Essays in Positive Economics. Chicago 1953.

HOUTHAKKER, Hendrik S., Can Speculators Forecast Prices? "The Review of Economics and Statistics", Cambridge, Mass., Vol. 39 (1957), p. 143-151.

KEMP, Murray C., Speculation, Profitability, and Price Stability. "The Review of Economics and Statistics", Cambridge, Mass., Vol. 45 (1963), p. 185-189.

KÖTHE, Gottfried, Topological Vector Spaces I. Translated by D.J.H. GARLING. New York 1969.

MILL, John Stuart, Principles of Political Economy. With some of their Applications to Social Philosophy. Vol. 2. 6th Ed. London 1865.

SCHIMMLER, Jörg, Speculation, Profitability, and Price Stability - a Formal Approach. "The Review of Economics and Statistics", Cambridge, Mass. Vol. 51 (1973), p. 110 - 114.

--------, Spekulation, spekulative Gewinne und Preisstabilitaet. Eine Theorie der Spekulation unter besonderer Berücksichtigung der Auswirkungen spekulativer Transaktionen auf die Preisstabilitaet. Meisenheim 1974.

TELSER, Lester G., A Theory of Speculation Relating Profitability and Stability. "The Review of Economics and Statistics", Cambridge, Mass., Vol. 41 (1959), p. 295-301.

Dr. Jörg Schimmler
Institut für Informatik und Praktische Mathematik
Universität Kiel
Olshausenstr. 40–60
2300 Kiel 1

Eingegangen am: 15.5.1975